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Abstracts of Papers to Appear in Future Issues

A GENERALISATION OF PLANAR MAGNETIC GRADIOMETER DESIGN VIA ORTHOGONAL POLYNOMIALS. A. E. Jones. *Department of Mathematics and Statistics, University of Paisley, Paisley, Renfrewshire, Scotland.* R. J. P. Bain. *Department of Physics and Applied Physics, University of Strathclyde, Glasgow, Scotland.*

We describe a problem in magnetic field detection involving a form of spatial filtering to detect weak signal sources in the presence of noise. Conventionally N-th order magnetic field gradiometers of fixed geometry are used in this situation. The pre-defined geometry completely determines the spatial sensitivity of such gradiometers. We demonstrate a method of making such devices much more flexible in that the near-source response can be modified while maintaining gradiometric order. The problem is described by the solution of N equations in sums and differences of powers, up to order N, of m variables, with $m > N$. The values of $(m - N)$ variables are chosen on physical considerations. We show that when values of the m variables are a solution set, they may be expressed as the roots of two polynomial equations, whose order is no greater than $(m + 1)/2$ when m is odd, or $m/2$ when m is even. These polynomial equations can be expressed as a linear combination of Chebyshev polynomials of the first and second kinds in the case of m odd and a related pair, fully described, in the case of m even. Existence of, and bounds on, solution sets are discussed and examples given.

COMPLEX KOHN VARIATIONAL PRINCIPLE FOR TWO-NUCLEON BOUND-STATE AND SCATTERING WITH THE TENSOR POTENTIAL. Carlos F. de Araujo, Jr., Sadhan K. Adhikari, and Lauro Tomio. *Instituto de Física Teórica, Universidade Estadual Paulista, 01405-000 São Paulo, São Paulo, Brazil.*

Complex Kohn variational principle is applied to the numerical solution of the fully off-shell Lippmann–Schwinger equation for nucleon–nucleon scattering for various partial waves including the coupled $^3S_1$–$^3D_1$ channel. Analytic expressions are obtained for all the integrals in the method for a suitable choice of expansion functions. Calculations with the partial waves $^1S_0$, $^1P_1$, $^1D_2$, and $^3S_1$–$^3D_1$ of the Reid soft core potential show that the method converges faster than other solution schemes not only for the phase shift but also for the off-shell t matrix elements. We also show that it is trivial to modify this variational principle in order to make it suitable for bound-state calculation. The bound-state approach is illustrated for the $^3S_1$–$^3D_1$ channel of the Reid soft-core potential for calculating the deuteron binding, wave function, and the D state asymptotic parameters.

SOLUTION OF HELMHOLTZ EQUATION IN THE EXTERIOR DOMAIN BY ELEMENTARY BOUNDARY INTEGRAL METHODS. S. Amini and S. M. Kirkup. *Department of Mathematics and Computer Science, University of Salford, Greater Manchester M5 4WT, United Kingdom.*

In this paper *elementary boundary integral equations* for the Helmholtz equation in the exterior domain, based on Green's formula or through representation of the solution by layer potentials, are considered. Even when the partial differential equation has a unique solution, for any given closed boundary Γ, these elementary boundary integral equations can be shown to be singular at a countable set of characteristic wavenumbers. Spectral properties and conditioning of the boundary integral operators and their discrete boundary element counterparts are studied near characteristic wavenumbers, with a view to assessing the suitability of these formulations for the solution of the exterior Helmholtz equation. Collocation methods are used for the discretisation of the boundary integral equations which are either of the Fredholm first kind, second kind, or hyper-singular type. The effect of quadrature errors on the accuracy of the *discrete collocation methods* is systematically investigated.

A CHEBYSHEV POLYNOMIAL INTERVAL-SEARCHING METHOD ("LANCZOS ECONOMIZATION") FOR SOLVING A NONLINEAR EQUATION WITH APPLICATION TO THE NONLINEAR EIGENVALUE PROBLEM. John P. Boyd. *Department of Atmospheric, Oceanic & Space Science, University of Michigan, 2455 Hayward Avenue, Ann Arbor, Michigan 48109, U.S.A.*

To search a given *real* interval for roots, our algorithm is to replace $f(\lambda)$ by $f_N(\lambda)$, its N-term Chebyshev expansion on the search interval $\lambda \in [\lambda_{min}, \lambda_{max}]$, and compute the roots of this proxy. This strategy is efficient if and only if $f(\lambda)$ itself is expensive to evaluation, such as when $f(\lambda)$ is the determinant of a large matrix whose elements depend nonlinearly on λ. For such expensive functions, it is much cheaper to search for zeros of $f_N(\lambda)$, which can be evaluated in $O(N)$ operations, than to iterate or look for sign changes on a fine grid with $f(\lambda)$ itself. It is possible to systematically increase N until the Chebyshev series converges acceptably fast, without wasting previously computed values of $f(\lambda)$, by imitating the Clenshaw–Curtis quadrature. Our strategy of replacing $f(\lambda)$ by $f_N(\lambda)$ is similar to Lanczos economization of power series, which also replaces an expensive function by a Chebyshev approximation that is more rapidly evaluated. The errors induced by the Chebyshev approximation can be eliminated by a final step of one or two iterations with $f(\lambda)$ itself, using the zeros of $f_N(\lambda)$ as initial guesses. We show through numerical examples that the algorithm works well. The only sour note is that it is sometimes necessary to split the search interval into subintervals, each with a separate Chebyshev expansion, when $f(\lambda)$ varies by many orders of magnitude on the search interval.

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